AP Calculus BC

WS 90 - Taylor/Maclaurin Series

1) $e^{-2x} = 1 + \frac{(-2x)}{1!} + \frac{(-2x)^2}{2!} + \frac{(-2x)^3}{3!} + \frac{(-2x)^4}{4!}$

$= 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4$

2) $\sin 2x = (2x) - \frac{(2x)^3}{3!}$

$= 2x - \frac{4}{3}x^3$

3) $\tan x = f(x)$

$f(0) = 0$

$f'(x) = \sec^2 x \quad f'(0) = 1$

$f''(x) = 2\sec x \cdot \sec x \tan x \quad f''(0) = 0$

$2\sec^2 x \tan x$

$f'''(x) = 2\sec^2 x \sec^2 x + 4\sec x \sec x \tan x \tan x$

$f'''(0) = 2$

$\tan x = x + \frac{2}{3!}x^3 = x + \frac{x^3}{3}$

4) e^x ; $a = 1$

$f(1) = e$

$f'(1) = e$

$f''(1) = e$

$f''(1) = e$

$T_3(x) = e + e(x-1) + \frac{e(x-1)^2}{2!} + \frac{e(x-1)^3}{3!}$

5) $f(x) = \ln x$; $a = 1$

$f(1) = 0$

$f'(x) = \frac{1}{x} \quad f'(1) = 1$

$f''(x) = -\frac{1}{x^2} \quad f''(1) = -1$

$f'''(x) = \frac{2}{x^3} \quad f'''(1) = 2$

$T_3(x) = (x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!}$

$= (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

6) $f(x) = \sqrt{x} \quad a = 4$

$f(4) = 2$

$f'(x) = \frac{1}{2\sqrt{x}} \quad f'(4) = \frac{1}{4}$

$f''(x) = -\frac{1}{4x^{3/2}} \quad f''(4) = -\frac{1}{32}$

$f'''(x) = \frac{3}{8x^{5/2}} \quad f'''(4) = \frac{3}{2^8}$

$T_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1/32}{2!}(x-4)^2 + \frac{3/2^8}{3!}(x-4)^3$

$= 2 + \frac{(x-4)}{4} - \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512}$

7) $e^{-x} = 1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \cdots$

$= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

8) $\frac{1}{1+x} = \frac{1}{1-(-x)} = 1+(-x)+(-x)^2+(-x)^3+\cdots$

$= 1 - x + x^2 - x^3 + \cdots =$

$= \sum_{n=0}^{\infty} (-1)^n x^n$

9) $f(x) = \frac{1}{x} \quad a = -1$

$f(-1) = -1$

$f'(x) = -\frac{1}{x^2} \quad f'(-1) = -1$

$f''(x) = \frac{2}{x^3} \quad f''(-1) = -2$

$f'''(x) = -\frac{6}{x^4} \quad f'''(-1) = -6$

$T_n(x) = -1 + -1(x+1) + \frac{-2}{2!}(x+1)^2 + \frac{-6}{3!}(x+1)^3 + \cdots$

$= -1 - (x+1) - (x+1)^2 - (x+1)^3 + \cdots$

$= \sum_{n=0}^{\infty} -(x+1)^n$

10) $f(x) = e^x$; $a = 2$

$f(2) = e^2$

$f'(2) = e^2$

$f''(2) = e^2$

$f'''(2) = e^2$

$T_n(x) = e^2 + e^2(x-2) + \frac{e^2}{2!}(x-2)^2 + \frac{e^2}{3!}(x-2)^3 + \cdots$

$= \sum_{n=0}^{\infty} \frac{e^2(x-2)^n}{n!}$

11) $f(x) = \ln x$; $a = 1$ SEE #5

$T_n(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(x-1)^n$